Buckling of Viscoelastic Beam Columns

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The paper presents a theoretical study of the creep buckling behavior of viscoelastic beam columns under general loading conditions. A detailed analysis is given for three particular cases, axially compressed columns with initial curvatures, laterally loaded beam columns, and beam columns in bending. The creep buckling problem is formulated in terms of the constitutive equations of the linear hereditary viscoelasticity. It includes two types of viscoelastic materials, those with limited and unlimited creep. The general solution is derived by means of the quasielastic method and is examined in detail for two simple rheological material models. It is shown that the creep buckling behavior of linearly viscoelastic beam columns under various loading conditions is typically governed by the magnitude of the axial compressive force. For viscoelastic materials of the limited creep type, there is a safe load limit below which the creep buckling characteristics of the structure are limited in time. The magnitude of the safe load limit as related to the Euler's elastic critical load depends solely on the asymptotic value of the creep function of the material. Quasielastic approximations and the corresponding exact analytical solutions are compared in two simple problems. It is observed that the quasielastic technique is adequate for applications to the linear creep buckling analysis.

Nomenclature

A_e	= amplitude of instantaneous deflection
A_i	= amplitude of initial imperfection
$A(t)$	= amplitude of lateral deflection
E_0	= instantaneous elastic modulus
I	= inertia moment
ℓ	= length of the beam column
M	= bending moment
M_0	= external bending moment
P	= axial force
P_e	= elastic critical load
q	= lateral pressure
q_0	= uniform lateral pressure
t	= time
w	= lateral deflection
w_e	= instantaneous elastic lateral deflection
w_i	= initial imperfection
Γ^*	= creep operator
$\Gamma(t-\tau)$	= creep kernel
ϵ	= strain
κ	= curvature
σ	= stress
ψ_∞	= limiting value of the creep function
$\psi(t)$	= creep function

Introduction

TIME-DEPENDENT buckling of columns in creep has been extensively studied in a number of publications.[1-15] Critical reviews on the subject are given by Distefano,[3] Kempner,[9] Vinogradov,[14] and Hoff.[16]

In general, the principal objectives of the creep buckling analysis can be outlined as follows: 1) to predict the critical time and 2) to obtain the viscoelastic buckling characteristics under various creep and loading conditions. The critical time in creep buckling is commonly associated with the development of large deformations at high deformation rates. Respectively, the creep buckling analysis should necessarily be based on the nonlinear deformation theory (see Huang,[7] Vinogradov,[14] and Zyczkowski[15]). The second case concerns primarily a variety of practical applications in which the creep deformations should not exceed a certain limit for the designed period of time. Such deformations are typically small and can be computed by means of the linear analysis.

The present paper deals with the latter class of problems. It aims at the creep buckling analysis of beam columns under general loading conditions, including as particular cases columns with initial imperfections under axial compression and beam columns subjected to simultaneous action of axial compressive forces, bending moments, and lateral pressure.

In the study, the creep properties of the material are defined by the constitutive equations of the linear viscoelastic theory of hereditary type. Formulation of the problem is given in general terms involving two types of viscoelastic materials—those with limited and unlimited creep. Typical characteristics of the viscoelastic buckling behavior are examined by means of the quasielastic method. Numerical results are obtained for two simple rheological material models. The accuracy of the quasielastic solution is discussed.

General Problem

Consider a hinge supported beam column subjected to simultaneous action of axial forces P, arbitrary distributed lateral pressure $q(x)$, and bending moments M_0, as shown in Fig. 1. The axis of the beam column may have an initial imperfection with the deviation $w_i(x)$ from the straight line. It is assumed that the magnitude of P is less than the elastic critical load P_e, $P < P_e$. The structure is loaded instantaneously at time $t = 0$, and the load is sustained at $t > 0$.

The mechanical properties of the material are defined by the constitutive equations of the linear viscoelastic theory of hereditary type. Thus, the uniaxial stress-strain relation is of the form

$$\epsilon(t) = \frac{1}{E^*}\{\sigma(t)\} = \frac{1}{E_0}(1+\Gamma^*)\{\sigma(t)\} \tag{1}$$

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where Γ^* is used as a symbolic notation for the integral operator of the Stieltjes convolution type

$$\Gamma^*\{\sigma(t)\} = \int_0^t \Gamma(t-\tau)\sigma(\tau)\,\mathrm{d}t \tag{2}$$

The kernel $\Gamma(t-\tau)$ represents the creep properties of the viscoelastic material.

It is assumed that the deformation of the column is governed by the classical Bernoulli-Euler theory. In this case, the curvature-bending moment relation is similar in form to Eq. (1):

$$\kappa = \frac{1}{E_0 I}(1+\Gamma^*)\{M\} \tag{3}$$

The curvature κ and the bending moment M are functions of the coordinate x and the time t.

With the above assumptions, the creep buckling behavior of the beam column is governed by the integrodifferential equation

$$E_0 I \frac{\partial^2 w}{\partial x^2} = -(1+\Gamma^*)\{M\} \tag{4}$$

in which the lateral deflection w is a function of x and t, $w = w(x,t)$. The governing equation (4) is considered in conjunction with the boundary conditions that are stationary in time

$$w(0,t) = w(\ell,t) = 0 \tag{5}$$

and the initial condition that represents the elastic response of the structure at the time of the load application

$$w(x,0) = w_e(x) \tag{6}$$

The viscoelastic buckling problem defined by Eqs. (4-6) is solved by means of the quasielastic method suggested by Schapery[17] for the linear viscoelastic stress analysis. The technique is similar to that applied by Vinogradov[14] to the creep buckling analysis of eccentrically compressed viscoelastic columns.

Application of the quasielastic method implies that the stress-strain relation in the form of integral equation (1) is approximated by

$$\epsilon(t) = \frac{1}{E_0}[1+\psi(t)]\sigma(t) \tag{7}$$

which is derived by replacing the action of the creep operator $\Gamma^*\{\sigma\}$ by the product $\psi(t)\cdot\sigma(t)$. In Eq. (7), $\psi(t)$ denotes the experimentally determined creep function of the material

$$\psi(t) = \Gamma^*\{1\} = \int_0^t \Gamma(t-\tau)\,\mathrm{d}t \tag{8}$$

Note that

$$\psi(0) = 0 \tag{9}$$

By means of the quasielastic method, the governing equation (4) to the viscoelastic buckling problem is replaced by

$$E_0 I \frac{d^2 w_q}{\mathrm{d}x^2} = -[1+\psi(t)]M \tag{10}$$

where $w_q \cong w$ denotes the quasielastic approximation of the lateral deflection w. The function w_q must satisfy the boundary and the initial conditions given by Eqs. (5) and (6), i.e.,

$$w_q(0,t) = w_q(\ell,t) = 0 \tag{11}$$

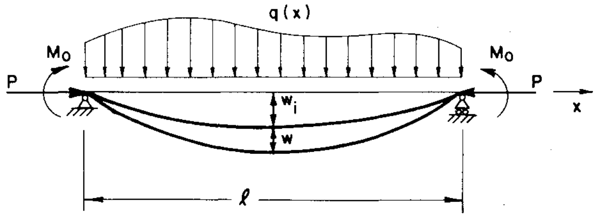

Fig. 1 Beam column with initial curvature.

$$w_q(x,0) = w_e(x) \tag{12}$$

Equation (12) indicates that at time $t=0$, the quasielastic approximation coincides with the exact viscoelastic solution.

Equation (10) can be presented in the equivalent form

$$E(t) I \frac{d^2 w_q}{\mathrm{d}x^2} = -M \tag{13}$$

where $E(t)$ denotes the time-dependent elastic modulus of a fictitious elastic material, the properties of which are defined by the actual viscoelastic response in terms of the creep function $\psi(t)$

$$E(t) = E_0/[1+\psi(t)] \tag{14}$$

It follows that the viscoelastic buckling problem is reduced to a similar elastic problem, which depends parametrically on time. Thus, the quasielastic approximation of the lateral deflection w_q can be readily obtained from the elastic analysis, provided that the creep properties of the structure and the loading conditions are specified.

Some Particular Cases

In order to examine the typical characteristics of the creep buckling behavior of viscoelastic beam columns, the above analysis is applied to several particular cases.

Columns with Initial Imperfections

Consider an initially bent viscoelastic column, the axis of which is defined as

$$w_i(x) = A_i \sin(\pi x/\ell) \tag{15}$$

The axial compressive force $P < P_e$ is applied to the column instantaneously at time $t=0$ and sustained for $t>0$. Respectively, the bending moment M is obtained as

$$M = P(w_i + w) \tag{16}$$

In this case, the governing equation (4) is of the form

$$E_0 I \frac{\partial^2 w}{\partial x^2} + P(1+\Gamma^*)\{w\} = -P(1+\Gamma^*)\{w_i\} \tag{17}$$

Replacing w_i by Eq. (15) and applying the quasielastic method, one arrives at the differential equation

$$\frac{d^2 w_q}{\mathrm{d}x^2} + k^2(t) w_q = -A_i k^2(t) \sin\frac{\pi x}{\ell} \tag{18}$$

where $k^2(t)$ is defined as

$$k^2(t) = k_e^2[1+\psi(t)] \tag{19}$$

$$k_e^2 = P/E_0 I \tag{20}$$

It can be verified by direct substitution that the quasielastic lateral deflection w_q in the form

$$w_q = A(t)\sin(\pi x/\ell) \tag{21}$$

satisfies Eq. (18), boundary conditions (11), and the initial condition (12). In Eq. (21), the amplitude $A(t)$ is defined as

$$A(t) = A_i \frac{\lambda[1+\psi(t)]}{1-\lambda[1+\psi(t)]} \tag{22}$$

where

$$\lambda = P/P_e \tag{23}$$

and the elastic critical load P_e is given by

$$P_e = \pi^2 E_0 I/\ell^2 \tag{24}$$

As follows from Eqs. (22) and (9), at the time of the load application $t=0$, the amplitude of the instantaneous elastic deflection is

$$A_e = A_i[\lambda/(1-\lambda)] \tag{25}$$

Respectively, the ratio $A(t)/A_e$ is obtained in the form

$$\frac{A(t)}{A_e} = \frac{(1-\lambda)[1+\psi(t)]}{1-\lambda[1+\psi(t)]} \tag{26}$$

This equation characterizes the creep buckling behavior of the axially compressed viscoelastic column with initial imperfection.

Beam Columns Under Lateral Pressure

Consider a viscoelastic beam column subjected simultaneously to the axial compressive force $P < P_e$ and the uniform lateral pressure q_0. In this case, the corresponding bending moment M is of the form

$$M = Pw + \tfrac{1}{2} q_0 x(\ell - x) \tag{27}$$

Following the solution procedure outlined above, Eq. (27) is introduced into the governing equation (4), which is subsequently treated by means of the quasielastic method. As the creep buckling problem under consideration is reduced to the corresponding time-dependent elastic problem, the elastic analysis can be employed. Thus, using the solution by Timoshenko,[18] one arrives at the quasielastic approximation of the amplitude of the lateral deflection in the form

$$A(t) = \frac{q_0 \ell^4}{8\lambda\pi^2 E_0 I}\left\{\frac{8}{\lambda[1+\psi(t)]\pi^2} \times \left[\frac{1}{\cos(\pi/2)\sqrt{\lambda[1+\psi(t)]}} - 1\right] - 1\right\} \tag{28}$$

This equation at $t=0$ yields the amplitude of the instantaneous elastic deflection A_e.

The ratio

$$\frac{A(t)}{A_e} = \frac{[8/1+\psi(t)]\left(\left\{1/\cos(\pi/2)\sqrt{\lambda[1+\psi(t)]}\right\} - 1\right) - \lambda\pi^2}{8\left[1/\cos(\pi/2)\sqrt{\lambda} - 1\right] - \lambda\pi^2} \tag{29}$$

represents the creep buckling behavior of the viscoelastic beam column subjected to axial compression and lateral pressure.

Beam Columns in Bending

Consider a viscoelastic beam column subjected to the axial compressive load $P < P_e$ and bending moments M_0 applied as shown in Fig. 1. In this case, the governing equation to the problem is of the form

$$E_0 I \frac{\partial^2 w}{\partial x^2} + P(1+\Gamma^*)\{w\} = -(1+\Gamma^*)\{M_0\} \tag{30}$$

Similarly to the previous cases, the quasielastic method is applied, and the result is obtained in terms of the time-dependent amplitude of the lateral deflection

$$A(t) = \frac{M_0 \ell^2}{\lambda\pi^2 E_0 I}\left\{2\frac{\sin(\pi/2)\sqrt{\lambda[1+\psi(t)]}}{\sin\pi\sqrt{\lambda[1+\psi(t)]}} - 1\right\} \tag{31}$$

It follows that the creep buckling characteristic of viscoelastic beam columns in bending is of the form

$$\frac{A(t)}{A_e} = \frac{2\left(\left\{\sin(\pi/2)\sqrt{\lambda[1+\psi(t)]}\right\}/\left\{\sin\pi\sqrt{\lambda[1+\psi(t)]}\right\}\right) - 1}{2\left\{\left[\sin(\pi/2)\sqrt{\lambda}\right]/\left[\sin\pi\sqrt{\lambda}\right]\right\} - 1} \tag{32}$$

Analysis of Results

The quasielastic solutions of the above-considered problems indicate that the viscoelastic buckling behavior of beam columns under various loading conditions is governed by two parameters, λ and $\psi(t)$. The first parameter represents the magnitude of the axial compression P as related to the elastic critical load P_e [see Eq. (23)]. Respectively, the second governing parameter $\psi(t)$ defines the creep properties of the structure.

According to Eqs. (26), (29), and (32), the creep buckling characteristics of beam columns increase in time and are either limited or unlimited, depending on the value of the product $\lambda[1+\psi(t)]$. Particularly, as $\lambda[1+\psi(t)] \to 1$, $A(t) \to \infty$.

The case of infinite deflections, however, is of little practical interest. Indeed, it refers to the situation conceptually inconsistent with the assumptions of the linear deformation theory. Clearly, the derived solutions are applicable only to the case of limited creep buckling for which

$$\lambda[1+\psi(t)] < 1 \tag{33}$$

Further in the analysis, two types of linear viscoelastic materials are distinguished, those exhibiting limited and unlimited creep. Limited creep is characterized by the creep function $\psi(t)$, which tends to a certain constant value ψ_∞ as $t \to \infty$. For materials with unlimited creep, $\psi_\infty = \infty$.

For viscoelastic materials with limited creep, the condition (33) is fulfilled at any time, even as $t \to \infty$, if the applied compressive load P does not exceed a certain value defined as a safe load limit P_s. The magnitude of P_s depends on the asymptotic behavior of the creep function of the material

$$P_s/P_e = 1/(1+\psi_\infty) \tag{34}$$

It follows that for materials with unlimited creep, the safe load limit $P_s = 0$. This result is identical with that obtained for initially bent viscoelastic columns by Distefano[3] and Kempner.[10] Similar behavior was observed from the creep buckling analysis of eccentrically loaded viscoelastic columns, circular arches, and spherical shells, see Vinogradov.[14,19,20]

A detailed analysis of the creep buckling behavior of beam columns can be afforded as the creep properties of the material

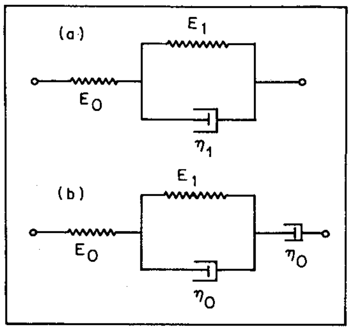

Fig. 2 Viscoelastic material models.

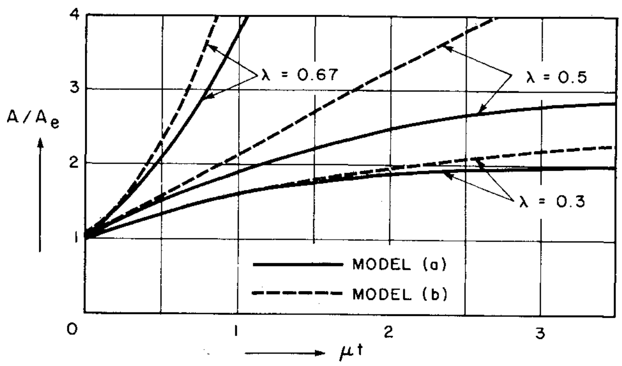

Fig. 3 Creep buckling characteristics.

are specified. Consider as an example two simple rheological material models shown in Fig. 2, which represent, respectively, materials with limited and unlimited creep. The model in Fig. 2a is known as a three-parameter standard solid with the creep function

$$\psi_{(a)}(t) = (E_0/E_1)(1 - e^{-\mu t}) \qquad \mu = E_1/\eta_1 \tag{35}$$

As $t \to \infty$, $\psi_{(a)}(\infty) = E_0/E_1$, i.e., the creep function $\psi_{(a)}$ is limited in time.

The model in Fig. 2b represents unlimited creep and is known as the Maxwell-Kelvin material with the creep function

$$\psi_{(b)}(t) = (E_0/E_1)(1 - e^{-\mu t}) + (E_0/\eta_0)t \tag{36}$$

As $t \to \infty$, $\psi_{(b)}(\infty) = \infty$.

For both material models, the elasticity and viscosity coefficients are specified as $E_0/E_1 = 0.5$ and $\eta_0/\eta_1 = 0.1$. According to Eq. (34), in the case of the three-parameter model, the safe load limit $P_s/P_e = 0.67$.

The creep buckling characteristics for three values of λ, $\lambda = 0.3$, 0.5, and 0.67 are computed using Eqs. (26), (29), and (32). The numerical results obtained in each of the above-considered problems are practically identical. These results are plotted vs the nondimensional time μt as shown in Fig. 3.

The analysis indicates that the lateral deflection of beam columns with limited creep, model a, is limited in time when the compressive load P does not exceed the safe load limit $P_s = 0.67$. Particularly, for $\lambda = 0.3$ and 0.5, the corresponding asymptotic values of the lateral deflections are $A(\infty)/A_e \cong 1.9$ and 3.0, respectively. For $\lambda \geq 0.67$ in the case of limited creep and for all magnitudes of λ in the case of unlimited creep, the structures tend to develop large deformations for which the condition (33) ceases to be valid at a certain point in time. It should be noted, however, that in the case of unlimited creep, the deflections may increase at low rates over extended time intervals if the values of λ are relatively small as compared with unity, i.e., $\lambda \leq 0.3$.

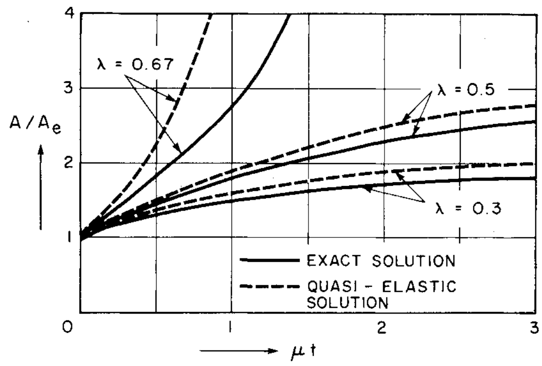

Fig. 4 Comparison of creep buckling characteristics (three-parameter standard solid).

Discussion

The approximate technique employed in this analysis entails the question as to the accuracy of the derived results. The studies by Schapery[17] and Amusin and Linkov[21] indicate that, in general, the quasielastic method provides adequate solutions to viscoelastic problems involving quasistatic loading conditions and low deformation rates. Applicability of this method to the creep buckling analysis of eccentrically compressed viscoelastic columns and circular arches is discussed by Vinogradov,[14,19] yielding the conclusion that the quasielastic approximations are sufficiently accurate when the loading parameters are below the safe load limit.

In this section, the accuracy of the quasielastic method is assessed by a comparison of the approximate results with the exact creep buckling characteristics given by Kempner[10] for some simple rheological material models. For axially compressed viscoelastic columns with initial imperfection in the form of Eq. (15), the exact analytical solution is of the form

$$\frac{A(t)}{\ell} = \frac{A_i}{\ell}\left(\frac{F(t)}{1-\lambda} - 1\right) \tag{37}$$

where the function $F(t)$ is specified as:

1) For the three-parameter standard solid,

$$F(t) = \frac{\lambda(E_0/E_1)}{\lambda(E_0/E_1) - 1 + \lambda} \times \left\{\exp\left[\left(\frac{\lambda}{1-\lambda}\frac{E_0}{E_1} - 1\right)\mu t\right] - 1\right\} + 1 \tag{38}$$

2) For the Maxwell-Kelvin material,

$$F(t) = \frac{1}{\omega_1 - \omega_2}\left[(\beta - \omega_2)e^{\omega_1 t} - (\beta - \omega_1)e^{\omega_2 t}\right] \tag{39}$$

in which

$$\beta = \frac{\lambda}{1-\lambda}E_0\left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right) \tag{40}$$

$$2\omega_{1,2} = \beta - \frac{E_1}{\eta_1} \pm \left[\left(\beta - \frac{E_1}{\eta_1}\right)^2 + \frac{4\lambda}{1-\lambda}\frac{E_0 E_1}{\eta_0\eta_1}\right]^{1/2} \tag{41}$$

The creep buckling characteristics are computed using the following numerical data $E_0/E_1 = 0.5$, $\eta_0/\eta_1 = 0.1$, and $\mu = E_1/\eta_1$ in Eqs. (37-41). Three values of the parameter λ are considered: $\lambda = 0.3$, 0.5, and 0.67. The obtained numerical results are presented in graphical form and compared with the corresponding quasielastic approximations as shown in Figs. 4 and 5.

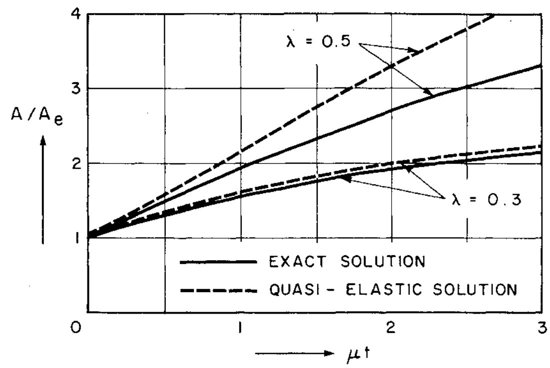

Fig. 5 Comparison of creep buckling characteristics (Maxwell-Kelvin model).

The diagrams indicate that for both material models the quasielastic method provides accurate results as the creep deformations remain relatively small. Particularly, this refers to the conditions below the safe load limit. Observing that the same conditions define the limits of applicability of the linear deformation theory, one arrives at the conclusion that the quasielastic method, in general, is adequate for applications to the linear creep buckling analysis.

Conclusions

The paper presents a consistent creep buckling analysis of beam columns under general loading conditions. Formulation of the problem is based on the constitutive equations of the linear viscoelastic theory and is given in terms of linear integral operators of the Stieltjes convolution type.

The analyses of three particular problems with different loading conditions indicate that the creep buckling behavior of viscoelastic beam columns is typically governed by the magnitude of the axial compressive force.

It is shown that for the viscoelastic materials with limited creep, there is a safe load limit below which the creep buckling characteristics are limited in time. The ratio of the safe load limit and the Euler's elastic critical load depends only on the long-term creep properties of the material.

In two simple cases, the approximate quasielastic solution of the creep buckling problem under consideration is compared with the exact viscoelastic analysis. This comparison indicates that the quasielastic method provides accurate results within the limits of applicability of the linear deformation theory.

Acknowledgments

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